Practice Exam 1

1. Perform the following integral.

$$\int \sec^5 x \tan^3 x \, dx$$

2. Perform the following integral.

$$\int xe^{-2x} \, dx$$

3. Evaluate the following integral

$$\int \frac{x}{\sqrt{x^2+9}} \, dx$$

4. Evaluate the following integral.

$$\int \frac{x+2}{x(x-1)^2} dx$$

5. Perform the following integral.

$$\int \frac{\sqrt{x^2-16}}{x} dx$$

6. Evaluate the following integral.

$$\int_0^{\pi/2} \sin^2 x \cos^3 x \, dx$$

7. Perform the following integral.

$$\int e^x \cos 3x\, dx$$

8. Perform the following integral.

$$\int \frac{3x-4}{(x+2)(x^2+4)}\, dx$$

11. $\int_0^\infty xe^{-x^2}dx$

12. $\int_{-\infty}^\infty \frac{1}{x^2+1}dx$

13. $\int_0^1 \dfrac{1}{\sqrt{1-x}} dx$

14. $\int_1^4 \dfrac{1}{x-3} dx$

15. Use the comparison theorem to determine whether $\int_2^\infty \dfrac{1}{\sqrt{x^2-1}} dx$ converges or diverges.

Answers:

1. $\dfrac{\sec^7 x}{7} - \dfrac{\sec^5 x}{5} + C$

2. $-\dfrac{1}{2}xe^{-2x} - \dfrac{1}{4}e^{-2x} + C$

3. $\sqrt{x^2 + 9} + C$

4. $2\ln|x| - 2\ln|x-1| - \dfrac{3}{x-1} + C$

5. $\sqrt{x^2 - 16} - 4\sec^{-1}\left(\dfrac{x}{4}\right) + C$

6. $\dfrac{2}{15}$

7. $\dfrac{e^x \cos 3x}{10} + \dfrac{3e^x \sin 3x}{10} + C$

8. $-\dfrac{5}{4}\ln|x+2| + \dfrac{5}{8}\ln|x^2 + 4| + \dfrac{1}{4}\tan^{-1}\left(\dfrac{x}{2}\right) + C$

11. ½

12. π

13. 2

14. Diverges

15. Let $g(x) = \dfrac{1}{\sqrt{x^2 - 1}}$ and $f(x) = \dfrac{1}{x}$.

By the P-theorem, $\displaystyle\int_2^\infty \dfrac{1}{x}\,dx$ diverges.

Also, $g(x) = \dfrac{1}{\sqrt{x^2 - 1}} \geq \dfrac{1}{\sqrt{x^2}} = \dfrac{1}{x} = f(x) \geq 0$ on the interval [2, ∞).

Thus by the Comparison Theorem, $\displaystyle\int_2^\infty \dfrac{1}{\sqrt{x^2 - 1}}\,dx$ diverges.